Embedding Impedance Approximations in the Analysis of SIS Mixers

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Abstract—Tucker's quantum theory of mixers is the basis for almost all analysis and design of SIS (Superconductor-Insulator-Superconductor) mixers. This paper examines the adequacy of three approximations to Tucker's theory: (i) the usual three-frequency approximation which assumes a sinusoidal LO voltage at the junction, and a short-circuit at all frequencies above the upper sideband, (ii) a five-frequency approximation which allows two LO voltage harmonics and five small-signal sidebands, and (iii) a quasi five-frequency approximation in which five small-signal sidebands are allowed, but the LO voltage is assumed sinusoidal. These are compared with a full harmonic-Newton solution of Tucker's equations, including eight LO harmonics and their corresponding sidebands, for realistic SIS mixer circuits.

It is shown that the accuracy of the three approximations depends strongly on the value of $\omega R_N C$ for the SIS junctions used. For large $\omega R_N C$, all three approximations approach the eight-harmonic solution. For $\omega R_N C$ values in the range 0.5 to 10, the range of most practical interest, the quasi five-frequency approximation is a considerable improvement over the three-frequency approximation, and should be suitable for much design work. For the realistic SIS mixers considered here, the five-frequency approximation gives results very close to those of the eight-harmonic solution.

I. INTRODUCTION

T HE SUPERIOR sensitivity of SIS mixer receivers at millimeter wavelengths has been clearly demonstrated in recent years [1], and SIS receivers are now used at almost all the world's major millimeter-wave radio astronomy observatories. SIS mixers appear for the most part to be well described by Tucker's quantum theory [2], [3], which predicts strongly non-classical behavior in a resistive mixer with an extremely sharp I–V nonlinearity. Tucker's theory is normally applied in its three-frequency approximation, in which form it is moderately tractable analytically and allows optimization of designs by small computer.

At present, the most sensitive SIS mixers have one or two mechanical waveguide tuners which allow the RF embedding impedance to be adjusted to suit the particular junction (or array of junctions). While mechanically tuned mixers are acceptable in applications requiring one or two receivers, plans for future radio astronomy instruments include arrays of many SIS receivers operating either all in the focal plane of a single
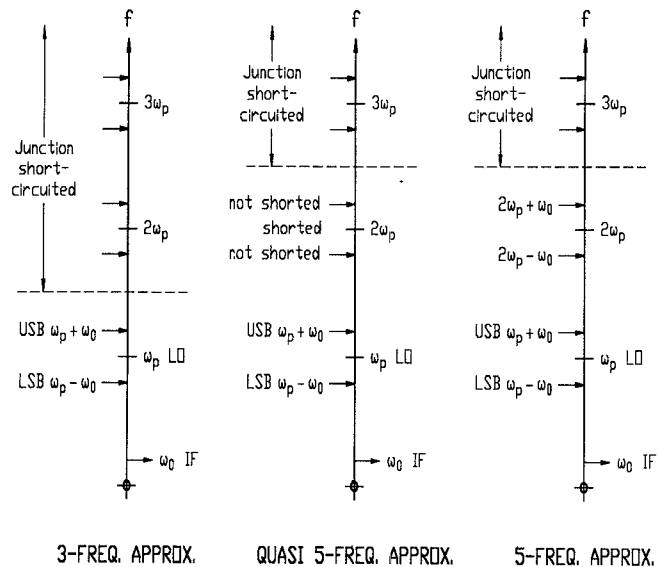
Fig. 1. Embedding impedance diagram indicating which frequencies are short-circuited for the three approximations: (a) three-frequency, (b) quasi five-frequency, and (c) full five-frequency.
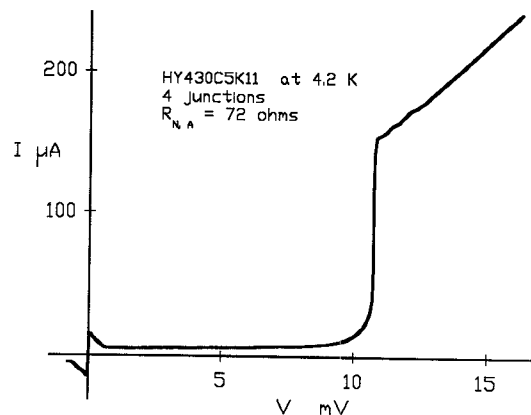

Fig. 2. I–V curve used in the simulations. This curve is for a four-junction array of Hypres Nb/Al-Al$_2$O$_3$/Nb junctions at 4.2 K, as used in [1].

radio telescope, or individually on the many antennas of a radio interferometer. For such applications, it is highly desirable to replace mechanically tuned mixers with broadband integrated mixers without mechanical tuners. To produce such mixers will require refinement of present mixer design techniques, most of which use Tucker's three-frequency approximation.

This paper examines the adequacy of three approximations to Tucker's theory: (i) the usual three-frequency approximation, (ii) a five-frequency approximation, and (iii) a quasi
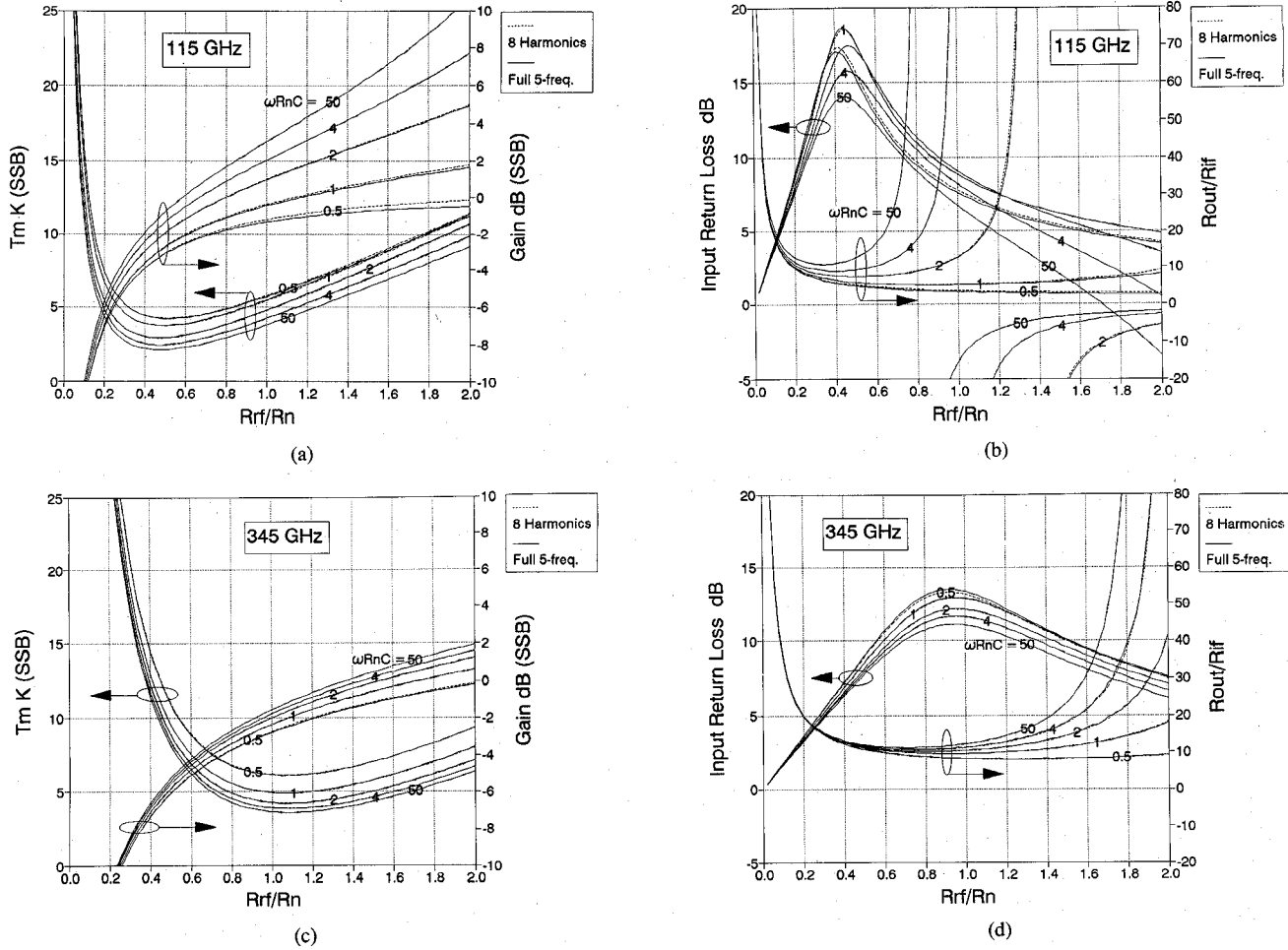
Fig. 3. Results using the full five-frequency approximation (solid lines) and eight LO harmonics (broken lines), shown as functions of R_{RF}/R_N, for $\omega R_N C = 0.5, 1, 2, 4, 50$. Output impedances are normalized to the IF load resistance R_{IF} (a) Noise temperature and conversion gain for a 115 GHz SIS mixer. (b) Input return loss and output impedance for a 115 GHz SIS mixer. (c) Noise temperature and conversion gain for a 345 GHz SIS mixer. (d) Input return loss and output impedance for a 345 GHz SIS mixer.

five-frequency approximation. These are compared with a full harmonic-Newton solution [4], [5] of Tucker's equations up to the eighth LO harmonic for realistic SIS mixer circuits. Use of the approximations greatly reduces the computational effort needed to optimize a mixer design.

II. THE APPROXIMATIONS

In the so-called three-frequency approximation to Tucker's theory, it is assumed, as indicated in Fig. 1(a), that the embedding impedance seen by the junction is finite at the LO frequency ω_p, and at ω_u, ω_l and ω_0, the upper and lower sideband and intermediate frequencies. At all higher frequencies the junction is short-circuited. This is likely to be a good approximation for junctions with large capacitance. The three-frequency approximation implies a sinusoidal LO voltage ($V_p \cos \omega_p t$) at the junction, for which case Tucker gives expressions for the elements $Y_{i,j}$ and $H_{i,j}$ of the mixer's 3×3 small-signal conversion and noise current correlation matrices as functions of the pumping parameter $\alpha = eV_p/\hbar\omega_p$ and dc bias voltage V_0.

The quasi five-frequency approximation assumes the same sinusoidal LO voltage waveform as above. However, while the

second LO harmonic $2\omega_p$ is short-circuited at the junction, the second harmonic sidebands $2\omega_p \pm \omega_0$ are not. This is depicted in Fig. 1(b). Expressions for the elements of the 5×5 small-signal conversion and noise current correlation matrices are given by Tucker. While this situation is not easy to implement practically, it is not physically unrealizable, and is expected to be a better approximation to a real mixer than the simple three-frequency approximation.

The full five-frequency approximation allows finite embedding impedances at all frequencies up to $2\omega_p + \omega_0$, but requires the junction to be short-circuited at all higher frequencies. This is depicted in Fig. 1(c). The LO voltage at the junction contains a second harmonic component, and an iterative algorithm must be used to determine the LO voltage waveform and thence the elements of the 5×5 small-signal conversion and noise current correlation matrices.

III. SIMULATIONS

The three approximations are investigated using hypothetical double sideband SIS mixers at 115 and 345 GHz, with the desired embedding impedances at the LO harmonics and
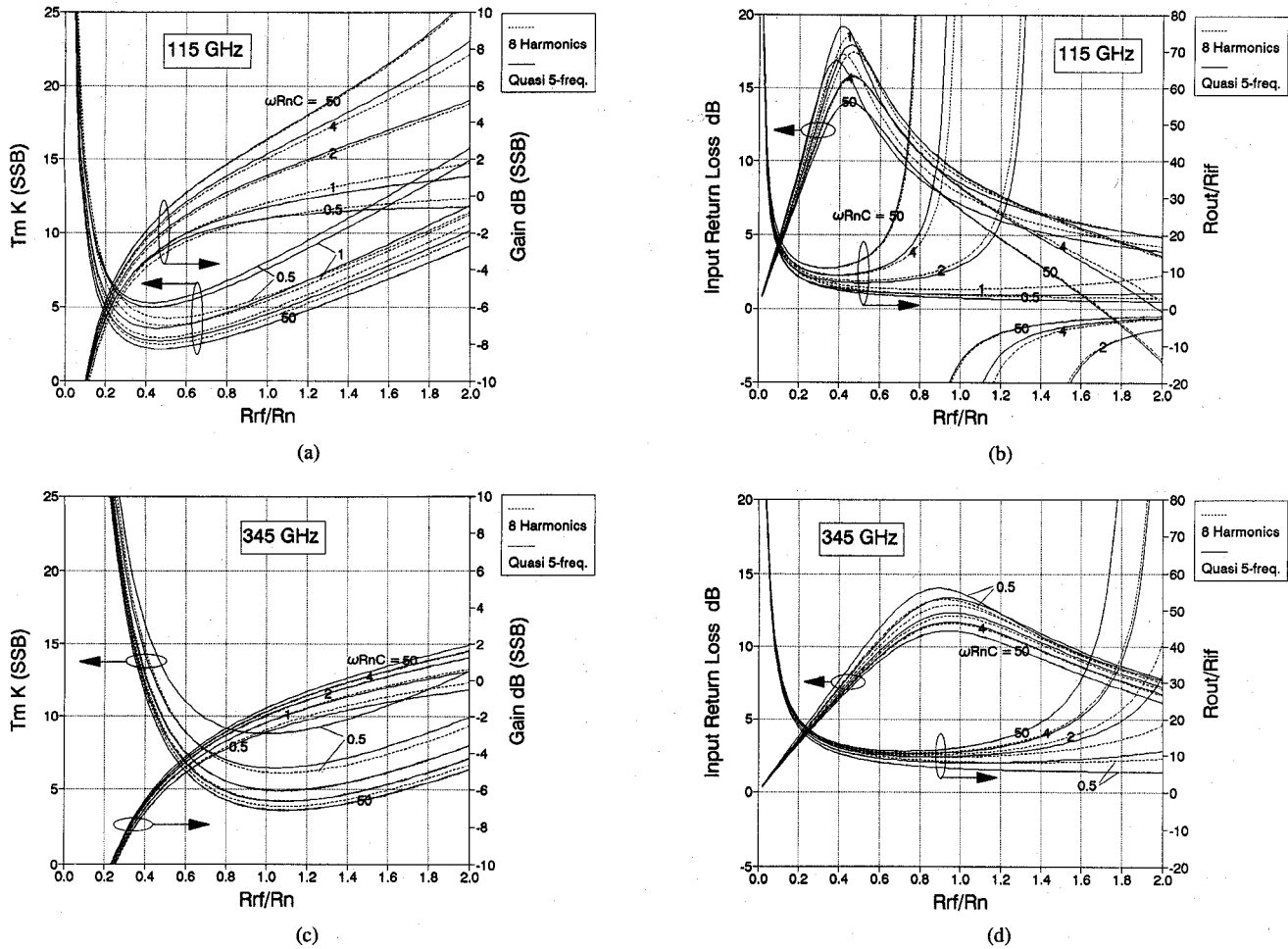
(a)

(b)

(c)

(d)

Fig. 4.   Results using the quasi five-frequency approximation (solid lines) and eight LO harmonics (broken lines), shown as functions of R_{RF}/R_N, for $\omega R_N C = 0.5, 1, 2, 4, 50$. Output impedances are normalized to the IF load resistance R_{IF}. (a) Noise temperature and conversion gain for a 115 GHz SIS mixer. (b) Input return loss and output impedance for a 115 GHz SIS mixer. (c) Noise temperature and conversion gain for a 345 GHz SIS mixer. (d) Input return loss and output impedance for a 345 GHz SIS mixer.

small-signal sideband frequencies. In all the examples, the junction capacitance is tuned out at frequencies ω_l and ω_u by the source susceptance. The embedding impedance at frequencies $\omega > \omega_p + \omega_0$ is either zero or that of the junction capacitance alone, depending on the particular example. The RF source and IF load conductances are assumed equal. These assumptions are consistent with realistic low noise mixer designs [1] with low IF $(\omega_0 \ll \omega_p)$.

The I–V curve used in the examples is that of the 4-junction array of Nb/Al-Al$_2$O$_3$/Nb junctions used in [1], and is shown in Fig. 2. The theoretical equivalence between a series array of junctions and a single junction is shown in [6].

Five values of $\omega_p R_N C$ are used: 0.5, 1, 2, 4, and 50. In all cases, the pumping parameter $\alpha = eV_p \hbar \omega_p = 1.2$, where V_p is the amplitude of the fundamental component of the LO voltage at the junction. The junction is voltage biased at the center of the first photon step below the gap voltage; i.e., $V_0 = V_{gap} - \hbar \omega_p / 2e$.

The mixer's (equivalent input) noise temperature, (transducer) conversion gain, input return loss, and output impedance are computed for each case as functions of R_{RF}/R_N, where R_{RF} is the reciprocal of the RF source conductance at the

signal frequency and R_N is the normal resistance of the junction (or array of junctions). For comparison, the results of the full eight-harmonic analysis are shown on each graph.

IV. RESULTS

Full Five-Frequency Approximation

The results of the full five-frequency approximation are shown in Figs. 3(a)–(d) for 115 GHz and 345 GHz mixers. At 115 GHz it is clear that the only significant deviation from the eight-harmonic results is in the gain for the lowest value of $\omega R_N C (0.5)$, and that is only a small fraction of a decibel. At 345 GHz there is almost no difference between the full five-frequency approximation and the eight-harmonic results.

Quasi Five-Frequency Approximation

Fig. 4(a)–(d) show the results of the quasi five-frequency analysis of the same mixers as above. At both 115 and 345 GHz, the gain is within a decibel of that computed using eight harmonics. The mixer noise temperatures agree closely for $\omega R_N C = 50$ and are within about 10% for $\omega R_N C = 4$, but
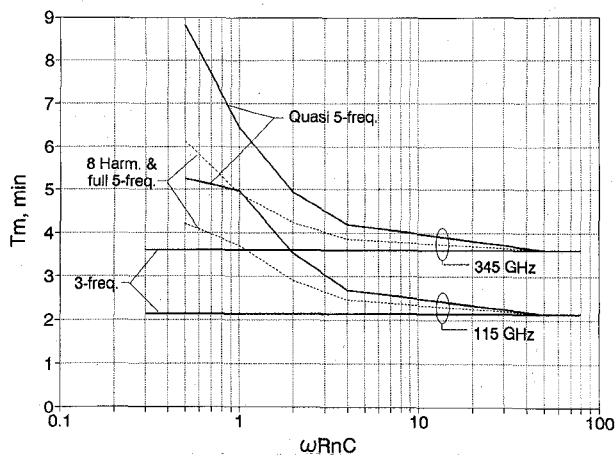
Fig. 5. Minimum mixer noise temperature as a function of $\omega R_N C$ for: (i) the full eight-harmonic and full five-frequency solutions (broken line), (ii) the quasi five-frequency approximation (solid line), and (iii) the three-frequency approximation (horizontal lines).

for $\omega R_N C = 0.5$ they differ by as much as 40%. The input return loss and output impedance show minor deviations from the eight-harmonic solution.

Three-Frequency Approximation

In the three-frequency case, the value of $\omega R_N C$ has no effect. This is because all relevant frequencies above the upper sideband are short-circuited, and we are assuming the junction capacitance is tuned out at the signal and image frequencies. The three-frequency case is thus equivalent to the high-$\omega R_N C$ limit of any of the other cases we have considered. The results of Figs. 3 or 4 with $\omega R_N C = 50$ are, in fact, indistinguishable from those of the three-frequency approximation.

V. DISCUSSION

In an earlier paper [1] we put forward design criteria for SIS mixers for millimeter-wave radio astronomy applications. They should have low noise, conversion gain near unity (to avoid saturation), and a reasonable RF input match. For wide frequency coverage, only double-sideband operation is considered. It was found that these conditions could be met if the mixer operated with equal RF source and IF load conductances, and that the output impedance of the mixer was then large (i.e., the mixer operates as a current source). The examples here likewise assume equal RF source and IF load conductances, and we have computed the mixer performance as a function of R_{RF}/R_N for the three embedding impedance approximations (R_{RF} is the reciprocal of the RF source conductance, and R_N is the normal resistance of the junction or array of junctions).

In designing a broadband tunerless SIS mixer, the choice of the value of $\omega R_N C$ is primarily a tradeoff between noise temperature and useable frequency range; large values of C will obviously limit frequency coverage, while too small an $\omega R_N C$ degrades the noise temperature as well as being more difficult to achieve without sacrificing junction quality. It is important, therefore, for the designer to have available a

method of mixer analysis which reflects the effect of $\omega R_N C$ with sufficient accuracy. From the results in Figs. 3 and 4, it is clear that the quasi five-frequency approximation describes the mixer performance quite well for $\omega R_N C \gtrsim 4$. For $0.5 \leq \omega R_N C < 4$, the gain, input return loss, and output impedance are well enough predicted by the quasi five-frequency approximation for most mixer design work, but the mixer noise temperature can be too high by as much as 40%. Fig. 5 shows the dependence of the minimum value of T_M (with respect to R_{RF}/R_N) on $\omega R_N C$ for each of the methods described here.

It may seem odd that the quasi five-frequency approximation should give quite accurate results for all the mixer parameters except the noise temperature. This is explained by considering the origin and nature of the noise at the output port of the mixer. This noise originates as shot noise from the current flowing in the SIS junction. The action of the local oscillator is, in effect, to amplitude modulate the shot noise produced by the DC current flowing in the junction, thereby generating correlated components at all the sideband frequencies $n\omega_p \pm \omega_0, n = 0, 1, \cdots$, etc. The mixing action of the time-varying junction conductance converts all these sideband components to the IF, preserving their correlation. The relative phase of each correlated IF component depends on the embedding impedance (including junction capacitance, at its sideband of origin, and on the phases of the harmonics in the LO waveform at the junction. The minimum noise occurs when these correlated IF noise components are phased so as to cancel to the greatest degree. (It is in this way that a classical diode mixer can, in principle, have zero noise temperature while operating with 3 dB conversion loss and many milliamps of rectified current.) The fact that, in low-noise operation, the SIS mixer has several quite large but strongly correlated output noise components which largely cancel one another, explains the sensitivity of the noise temperature results to the small changes in LO waveform between the quasi five-frequency and full five-frequency approximations. A more detailed discussion of noise conversion in mixers, and additional references, are given in [7].

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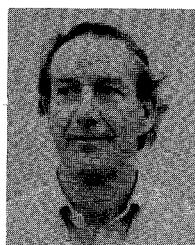
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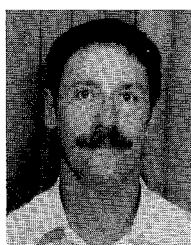
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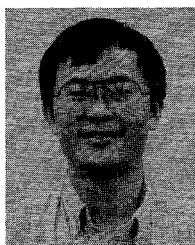
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